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LETTER TO THE EDITOR

Monte Carlo study of a vortex glass model

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Abstract. We describe results of Monte Carlo simulations on a model that seems to have the necessary ingredients to describe a disordered type-II superconductor in a magnetic field. We compute the free energy cost to twist the direction of the phase of the condensate and analyse the results by finite-size scaling. The results show convincingly that the model has different behaviour as a function of dimension: in $d=4$ the model clearly has a finite transition temperature; T_c , while for $d=2$ only there is only a transition at $T=0$.

Since fluctuation effects play a much more important role in high-temperature superconductors than in conventional superconducting materials, there has been a great deal of effort [1] to understand the behaviour of type II superconductors in a magnetic field, including the effects of disorder, when one goes beyond the mean field picture of BCS or Ginzburg–Landau theories. One intriguing aspect which has emerged is the possibility of a vortex glass phase [2, 3] in which the off-diagonal long-range order of the pair condensate has a phase which is random in space but frozen in time, much like the order parameter in a spin glass [4]. This can arise because the Abrikosov flux lattice, which forms in pure samples, is destroyed by disorder in less than four dimensions [5] beyond a certain length scale, l_{dis} . The phase of the condensate does not then form a regular periodic pattern on scales larger than l_{dis} , but, according to the vortex glass hypothesis, the system undergoes a transition into a spin glass-like state in which the phase is frozen in time. At the transition, the vortex glass correlation length, ξ diverges. A number of experiments [6] have found evidence for such a transition in the I–V characteristics of Y–B–Cu–O samples. Only if there is a vortex glass phase does the resistance really vanish [1] for $H > H_{c1}$. Otherwise, the resistance is, in principle, finite because clusters of vortices on scale ξ can move by thermal activation over barriers, a process known as ‘flux creep’ [7]. These effects are observable [8] in high- T_c compounds since they have much larger fluctuations than conventional materials.

In this letter we describe results of Monte Carlo simulations, analysed by finite-size-scaling techniques, on a model that seems to have the necessary ingredients to describe the vortex glass state. The results show convincingly the different behaviour occurring in different dimensions. In $d=4$ there is clearly a transition at finite transition temperature T_c with vortex glass order at lower temperature, while in $d=2$ there is only a transition at $T=0$. Analogous results for $d=3$ have been presented

before [9], and indicate behaviour close to what is expected at the lower critical dimension, though with probably a finite T_c . Our results for $d=4$ have been briefly described in a conference proceeding [10]. We feel that it is useful to include them here as a contrast to the new results for $d=2$ to emphasize the power of the finite-size-scaling technique in elucidating whether or not a glass-like transition occurs.

The model that we study, known as the 'gauge glass', has the following Hamiltonian:

$$\mathcal{H} = - \sum_{\langle i, j \rangle} \cos(\phi_i - \phi_j - A_{ij}). \quad (1)$$

The phase, ϕ_i , is defined on each site of a regular lattice, square for two dimensions, simple cubic for $d=3$ and simple hypercubic for $d=4$, with $N=L^d$ sites. Periodic boundary conditions are imposed. The sum is over all nearest neighbour pairs on the lattice. The effects of the magnetic field and disorder are represented by the quenched vector potentials, A_{ij} , which we take to be independent random variables with a uniform distribution between 0 and 2π . This model seems to be the simplest one with the correct ingredients of randomness, frustration and order parameter symmetry. It does, however, ignore screening, and therefore corresponds to an extreme type II limit in which $\kappa = \lambda/\xi \rightarrow \infty$, where λ is the penetration length. Since $\kappa \gg 1$ in the high T_c superconductors, this limit is not unreasonable. It is unclear, however, how much inclusion of screening via a fluctuating gauge field would modify the behaviour of equation (1).

If the A_{ij} are restricted to the values 0 and π , the model becomes the XY spin glass, for which the lower critical dimension is believed [11] to be four. However, earlier work [13, 12, 9], has shown that the gauge glass is in a different universality class from the XY spin glass, presumably because it does not have the 'reflection' symmetry, $\phi_i \rightarrow -\phi_i \forall i$ [13].

As discussed before [9, 12], it is useful to consider the change in free energy ΔF when one imposes a twist Θ along one of the space directions, x say. More precisely, the periodic boundary conditions, $\phi_i = \phi_{i+Lx}$ are replaced by the twisted boundary conditions, $\phi_i = \phi_{i+Lx} + \Theta$. By a simple redefinition of the phases ϕ_i one can replace this situation by a system with periodic boundary conditions and an extra contribution, Θ/L , to the vector potential on bonds in the x -direction.

By Monte Carlo methods one can calculate derivatives of the free energy w.r.t. Θ , so, for a single sample, we define a current, I , and a stiffness, Y , by

$$I \equiv \frac{\partial F}{\partial \Theta} = \frac{1}{L} \sum_i \langle \sin \Delta_i \rangle_T \quad (2)$$

$$Y \equiv \frac{\partial^2 F}{\partial \Theta^2} = \frac{1}{L^2} \left\{ \sum_i \langle \cos \Delta_i \rangle_T - \frac{1}{T} \sum_{i,j} [\langle \sin \Delta_i \sin \Delta_j \rangle_T - \langle \sin \Delta_i \rangle_T \langle \sin \Delta_j \rangle_T] \right\} \quad (3)$$

where $\Delta_i = \phi_i - \phi_{i+x} - A_{i,i+x}$, F is the total free energy and $i+x$ refers to the nearest neighbour site in the x -direction from i . Note that both I and Y are gauge invariant so they are still useful even if one includes fluctuating gauge fields.

Above T_c , ΔF , and hence both I and Y , go to zero rapidly with increasing system size because the system is insensitive to boundary conditions when L is much greater than the vortex glass correlation length ξ . If T_c is finite, then, below T_c , I and Y vary

with L as L^θ where $\theta (>0)$, is an exponent describing the low temperature phase. In other words, I and Y increase with increasing L below T_c , the opposite of what happens above T_c . Precisely at T_c , both I and Y are independent of size. Hence if T_c is finite, I and Y should come together at T_c and splay out again at lower temperatures. By contrast, if $T_c=0$, then, at $T=0$, I and Y vary as L^θ but with $\theta < 0$. Consequently, I and Y decrease with L even at $T=0$.

In a disordered system, it is necessary to perform an average over different realizations of the disorder, which we indicate by $[\dots]_{av}$. For the gauge glass, the average values of I and Y are both zero, i.e.

$$[Y]_{av} = [I]_{av} = 0 \quad (4)$$

because the configuration in which the vector potentials in the x direction have been increased by Θ/L has the same weight in the configurational average as the original choice of vector potentials. One is therefore interested in the root mean square fluctuation between samples. This means that many samples must be averaged over typically several thousand. If T_c is finite, the finite size scaling form for the r.m.s. current, ΔI is therefore

$$\Delta I = [I^2]_{av}^{1/2} = \bar{I}(L^{1/\nu}(T - T_c)) \quad (T_c > 0) \quad (5)$$

where ν is the correlation length exponent. We shall concentrate on the r.m.s. current in what follows, rather than the stiffness, because sample to sample fluctuations in the stiffness have an asymmetric distribution with a long tail, which makes it difficult to get good statistics [9]. If $T_c=0$ then ΔI decreases with size even at $T=0$, i.e. $\Delta I \sim L^\theta$ where θ is negative and related to the exponent ν giving the divergence of the

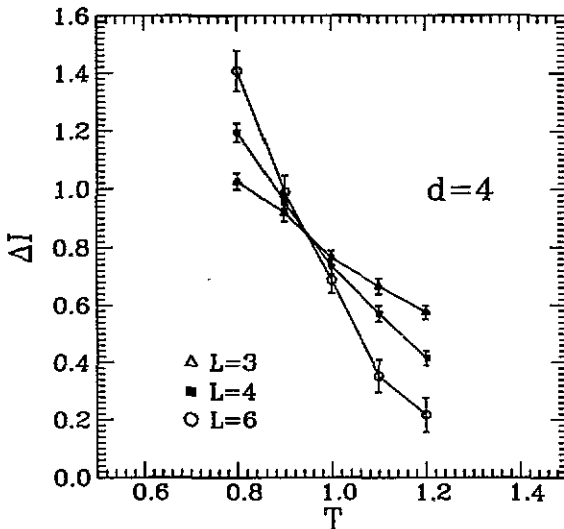


Figure 1. The r.m.s. current, $\Delta I = [I^2]_{av}^{1/2}$, for $d=4$ determined by Monte Carlo simulations for different sizes and temperatures. The curves for different sizes are expected to come together at T_c and, if there is order in the low temperature state, to splay out again at lower temperatures. The data does indeed follow this behaviour.

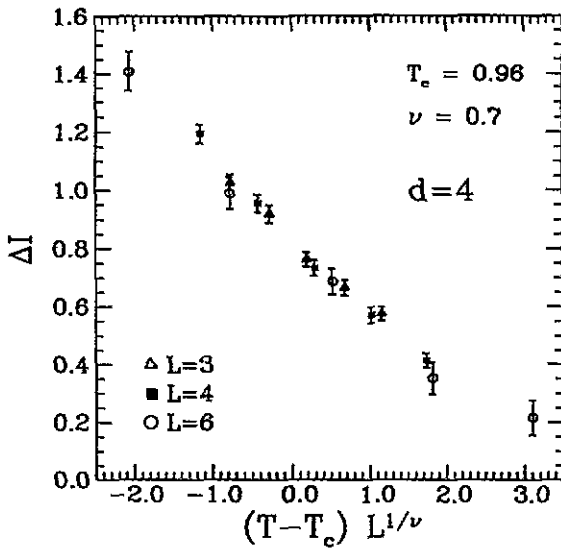


Figure 2. The same data as in figure 1 but in a finite size scaling plot, with $T_c = 0.96$ and $\nu = 0.7$.

correlation length as $T \rightarrow 0$ by $-\theta = 1/\nu$. The finite size scaling form is then

$$L^{1/\nu} \Delta I = \tilde{f}(L^{1/\nu} T) \quad (T_c = 0). \quad (6)$$

Tests to ensure equilibration were carried out as described elsewhere [14].

We first show results for $d=4$, [10]. Figure 1 shows clearly that the data for the r.m.s. current for different sizes come together at $T = T_c \approx 0.95$ and then splay out again on the low temperature side, just as expected at a finite temperature transition.

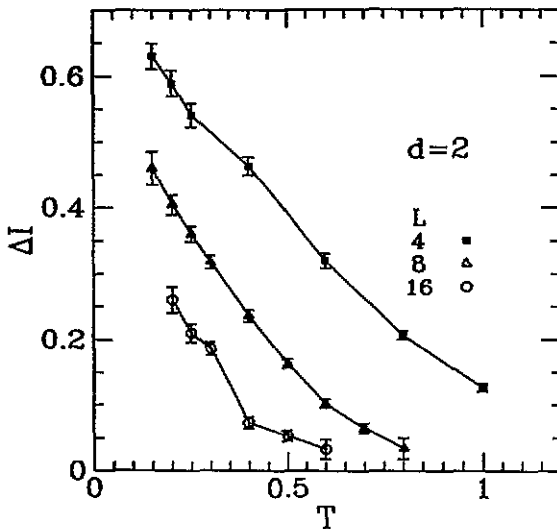


Figure 3. The r.m.s. current, $\Delta I = [I^2]_{av}^{1/2}$, for $d=2$ determined by Monte Carlo simulations for different sizes and temperatures. The curves for different sizes do not come together, even at the lowest temperature. This behaviour indicates a transition at $T=0$.

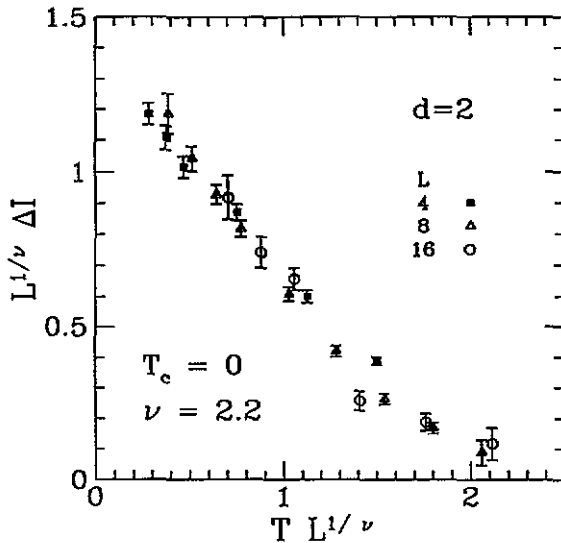


Figure 4. The same data as in figure 4 but in a finite size scaling plot, with $T_c=0$ and $\nu=2.2$.

This provides unambiguous evidence that there is a vortex glass transition in four dimensions (and presumably also in higher dimensions) with vortex-glass order on the low-temperature side. A scaling plot corresponding to (5) is shown in figure 2. From the fit we estimate

$$T_c = 0.96 \pm 0.01 \quad \nu = 0.7 \pm 0.15 \quad (d=4). \quad (7)$$

Next we discuss the case of $d=2$. The results for ΔI are shown in figure 3. Notice that they are quite different from figure 1, since, even at the lowest temperature, ΔI decreases with increasing size. This is precisely what is expected at a zero temperature transition, and the scaling plot in figure 4 corresponding to (6) works very well. From the fit we estimate

$$T_c = 0 \quad \nu = 2.2 \pm 0.2 \quad (d=2). \quad (8)$$

The value for ν agrees with earlier work [12], in which a different finite-size-scaling technique was used. Recent experiments [15] on very thin (16 Å) films of YBCO have provided striking confirmation that $T_c=0$ for the vortex glass in two dimensions. Furthermore, it is found that nonlinear current-voltage characteristics set in when the current density exceeds a value, J_{nl} which, according to scaling theory [12] varies with temperature as $T^{1+\nu}$. According to conventional flux creep theory, there is no divergent length scale as $T \rightarrow 0$, which corresponds to setting $\nu=0$. The experiments [15] find $1+\nu=3.0 \pm 0.3$, in excellent agreement with the results presented here and in [12], but in clear disagreement with the flux creep theory. We emphasize, then, that the vortex glass picture leads to measurable consequences even when $T_c=0$.

To conclude, we have shown that a Monte Carlo calculation of the change in free energy due to a twist in the boundary conditions combined with finite size scaling is a very powerful tool for systems with XY-like symmetry. We have shown that it clearly distinguishes between the finite temperature transition in the $d=4$ gauge glass and the zero temperature transition of the two-dimensional model.

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